AP Calculus BC

Linearization

1) $f(x) = x^3 - 2x + 2$, $a = 2$

a) $f(2) = 6$

$f'(x) = 3x^2 - 2$

$f'(2) = 10$

$L(x) = 6 + 10(x-2)$

b) $f(2.1) \approx L(2.1) = 6 + 10(2.1-2)$

$= 6 + 10(0.1)$

$= 7$

Error $= \left| L(2.1) - f(2.1) \right| = 0.061$

Percent Error $= \dfrac{(0.061)(100)}{|f(2.1)|} = 0.86\%$

2) $f(x) = \sqrt[3]{x}$, $a = 8$

a) $f(8) = 2$

$f'(x) = \frac{1}{3} x^{-2/3}$

$f'(8) = \frac{1}{12}$

$L(x) = 2 + \frac{1}{12}(x-8)$

b) $f(8.1) \approx L(8.1) = 2 + \frac{1}{12}(0.1) = 2 + \frac{1}{120} = 2\frac{1}{120}$

Error $= \left| L(8.1) - f(8.1) \right| = 0.000034$

% Error $= \dfrac{0.000034}{|f(8.1)|} \times 100 = 0.0017\%$

3) $f(x) = (1+x)^k$

$f(0) = 1$

$f'(x) = k(1+x)^{k-1}$

$f'(0) = k$

$L(x) = 1 + k(x-0) = 1 + kx$

4) $f(x) = \sqrt{x}$ $a = 100$

$f(100) = 10$

$f'(x) = \frac{1}{2\sqrt{x}}$

$f'(100) = \frac{1}{20}$

$L(x) = 10 + \frac{1}{20}(x-100)$

$f(101) \approx L(101) = 10 + \frac{1}{20}(101-100)$

$= 10 + \frac{1}{20}(1)$

$= \frac{201}{20}$

5) $f(x) = x^3 + 3x$

$f(2) = 14$

$f'(x) = 3x^2 + 3$

$f'(2) = 15$

$L(x) = 14 + 15(x-2)$

$f(2.01) \approx L(2.01) = 14 + 15(2.01 - 2)$

$= 14.15$

6) $f(x) = \sqrt{x} + x^2$ $a = 25$

$f(25) = 630$

$f'(x) = \frac{1}{2\sqrt{x}} + 2x$

$f'(25) = 50.1$

$L(x) = 630 + 50.1(x-25)$

$f(24.9) \approx L(24.9) = 630 + 50.1(-0.1)$

$= 630 - 5.01$

$= 624.99$

7) $f' = 2x+1 \quad f(1) = 4$

$f'(1) = 3$

$L(x) = 4 + 3(x-1)$

$f(1.2) \approx L(1.2) = 4 + 3(0.2)$

$= 4.6$

8) $f(x) = \sqrt{x^2+9} \quad a = -4$

$f(-4) = 5 \qquad f'(x) = \dfrac{x}{\sqrt{x^2+9}}$

$f'(-4) = -\frac{4}{5}$

$L(x) = 5 - \frac{4}{5}(x+4)$

$f(-3.9) \approx L(-3.9) = 5 - 0.8(0.1)$

$= 5 - 0.08$

$= 4.92$

9) $f(x) = x^{4/3} \quad a = 8$

$f(8) = 16 \qquad f'(x) = \frac{4}{3}x^{1/3}$

$f'(8) = 16/3$

$L(x) = 16 + 16/3(x-8)$

$f(8.4) \approx L(8.4) = 16 + \frac{16}{3}\left(\frac{2}{5}\right)$

$= 16 + \frac{32}{15}$

10) $\left.\dfrac{dy}{dx}\right|_{(1,-2)} = 12 - 56 = -44$

$L(x) = -2 - 44(x-1)$

$f(1.3) \approx L(1.3) = -2 - 44(0.3)$

Since the graph of $f(x)$ is concave up, $f(1.3) > L(1.3)$. The approximation is an underestimate

11) $g(3) = 17 \quad g'(3) = -1.2$

$L(x) = 17 - 1.2(x-3)$

$f(3.2) \approx L(3.2) = 17 - 1.2(0.2)$

$= 17 - \frac{6}{5}\left(\frac{1}{5}\right)$

$= 17 - \frac{6}{25}$